**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Soln: *μ* = 45 minutes

*σ* = 8 minutes

Let X be the amount of time it takes to complete the servicing work.

To finish in 1 hour, X < 50 since the work was started 10 minutes after the car is dropped off.

Z = (X - *μ )/ σ*

= (50 – 45)/8

= 0.625

From Z score table, we get P (X>50) = 0.2676.

So the probability that the service manager cannot meet his commitment is 0.2676.

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Soln: *μ* = 38

*σ* = 6

Let X be the number of employees.

1. Probability of employees older than 44 is P (X>44) = 1 – P (X≤44)

Z = (X - *μ )/ σ*

= (44 – 38)/6

= 1

From Z score table, we get P (X≤44) = 0.1586.

P (X>44) = 1 – P (X≤44)

= 1 – 0.1586

= 0.8414

From above value, we can say that 84.14% employees are older than 44.

Employees between the age of 38 and 44 are 1 – 0.8414 i.e., 0.1586.

15.86% employees are between 38 and 44.

So the statement More employees at the processing center are older than 44 than between 38 and 44 is true.

1. Probability of employees under the age of 30 is P (X<30)

Z = (X - *μ )/ σ*

= (30 – 38)/6

= - 1.33

From Z score table, we get P (X<30) = 0.0917

9.17% of employees are under the age of 30.

So the number of employees under the age of 30 is 0.0917 \* 400 =36.68 (approx. 36)

Hence the statement A training program for employees under the age of 30 at the center would be expected to attract about 36 employees is also true.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Soln:

As we know that if X ∼ N(µ1, σ1^2 ), and Y ∼ N(µ2, σ2^2 ) are two independent variables then X + Y ∼ N(µ1 + µ2, σ1^2 + σ2^2 ) , and X − Y ∼ N(µ1 − µ2, σ1^2 + σ2^2 ).

Similarly if Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X and Y , then Z ∼ N(aµ1 + bµ2, a^2σ1^2 + b^2σ2^2 ).

2X1~ N(2 u,4 σ^2) and

X1+X2 ~ N(µ + µ, σ^2 + σ^2 ) ~ N(2 u, 2σ^2 )

2X1-(X1+X2) = N( 4µ,6 σ^2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Soln:

Since we need to find out the values of a and b, which are symmetric about the mean, such that the probability of random variable taking a value between them is 0.99, we have to work out in reverse order.

The Probability of getting value between a and b should be 0.99.

So the Probability of going wrong, or the Probability outside the a and b area is 0.01 (ie. 1-0.99).

The Probability towards left from a = -0.005 (ie. 0.01/2).

The Probability towards right from b = +0.005 (ie. 0.01/2).

So since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.

By finding the Standard Normal Variable Z (Z Value), we can calculate the X values.

Z=(X- μ) / σ

For Probability 0.005 the Z Value is -2.57 (from Z Table).

Z \* σ + μ = X

Z(-0.005)\*20+100 = -(-2.57)\*20+100 = 151.4

Z(+0.005)\*20+100 = (-2.57)\*20+100 = 48.6

So, option D is correct.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Soln:

Mean profits from two different divisions of a company = Mean1 + Mean2

Mean **=** 5**+**7

Print ('Mean Profit is Rs', Mean**\***45,'Million')

Mean Profit is Rs 540 million.

Variance of profits from two different divisions of a company = SD^2 = SD1^2 + SD2^2

SD **=** np**.**sqrt((9)**+**(16))

Print ('Standard Deviation is Rs', SD**\***45, 'Million')

Standard Deviation is Rs 225.0 million

1. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Print ('Range is Rs',(stats**.**norm**.**interval(0.95,540,225)),'in Millions')

Range is Rs (99.00810347848784, 980.9918965215122) in Millions

B. Specify the 5th percentile of profit (in Rupees) for the company

# To compute 5th Percentile, we use the formula X=μ + Zσ; wherein from z table,

5 percentile = -1.645

X**=** 540**+**(**-**1.645)**\***(225)

print('5th percentile of profit (in Million Rupees) is',np**.**round(X,))

5th percentile of profit (in Million Rupees) is 170.0

C. Which of the two divisions has a larger probability of making a loss in a given year?

Probability of Division 1 making a loss P(X<0)

stats**.**norm**.**cdf(0,5,3)

0.0477903522728147

Probability of Division 2 making a loss P(X<0)

stats**.**norm**.**cdf(0,7,4)

0.040059156863817086

Hence probability of Division 1 has a larger probability of making a loss in a given year.